MI-0195

# Reference Orbit Trajectory in a Combined Function Magnet

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The Recycler is to be constructed utilizing combined function magnets. Since these magnets are to be fabricated without sagitta, the reference orbit will not follow a trajectory of constant field and thus will not be the arc of a circle. The purpose of this note is to calculate the reference orbit through the Recycler combined function magnets.

## Reference Orbit

The motion for the reference orbit within a combined function magnet is approximated by the differential equation,

$$\frac{d^2x}{dz^2} \pm \left(\frac{B'}{B\rho}\right) x = \left(\frac{B_0}{B\rho}\right) \tag{1}$$

where ($B\rho$) is the magnetic rigidity, $B_0$ is the field measured at x=0, and B' is the field gradient at x=0. The + and - signs refer to the motion in a focusing and defocusing magnet respectively. The coordinate system used here is Cartesian: z measures position along the longitudinal magnet axis (not along the reference orbit), and x measures position perpendicular to z and to the magnetic field. Contributions to the field of higher order than linear are insignificant for Recycler magnet parameters and are ignored here.

Equation (1) can be solved in terms of the value of $x_0$ and $x_0'$ at the magnet entrance:

$$\begin{cases} x(z) = (x_0 + \frac{B_0}{B'}) \cos(\sqrt{k}z) + \frac{x_0'}{\sqrt{k}} \sin(\sqrt{k}z) - \frac{B_0}{B'} \\ x'(z) = -\sqrt{k}(x_0 + \frac{B_0}{B'}) \sin(\sqrt{k}z) + x_0' \cos(\sqrt{k}z) \end{cases} \tag{2}$$

$$\begin{cases} x(z) = (x_0 + \frac{B_0}{B'}) \cosh(\sqrt{k}z) + \frac{x_0'}{\sqrt{k}} \sinh(\sqrt{k}z) - \frac{B_0}{B'} \\ x'(z) = \sqrt{k}(x_0 + \frac{B_0}{B'}) \sinh(\sqrt{k}z) + x_0' \cosh(\sqrt{k}z) \end{cases} \tag{3}$$

where $k = B'/(B\rho)$ and equations (2) and (3) apply to focusing and defocusing magnets respectively.

It is instructive to compare the trajectories described by (2) and (3) in a Recycler gradient magnet to the trajectories described by an arc of a circle. A comparison is shown in the figure below for the 4.2 meter Recycler focusing magnet (nominal bend radius of 201.4 meters). Here it assumed that the magnet is aligned so that the entrance angle is one half the total nominal bend angle (10.43 mrad) and the entrance position is one-half the nominal sagitta (-5.4 mm).

Little deviation between the real reference orbit and the circular trajectory is evident when view on a figure at this scale. However, a close examination reveals that the reference trajectory has rotated through an additional 90 μr in the magnet and that the output position is displaced by 0.2 mm. This is due to the additional bending experienced on the reference orbit as it moves to the outside of the (focusing) magnet. The situation is reversed in the defocusing magnets. The over (under) bending is significant, corresponding to about 40 "units" of strength deviation. If all magnets were built to a nominal integrated strength as measured along the z axis the effect on the closed orbit would be significant. This particular effect can be ameliorated either by displacing the magnets transversely (~2 mm) or by targeting the strength differently for F and D magnets.

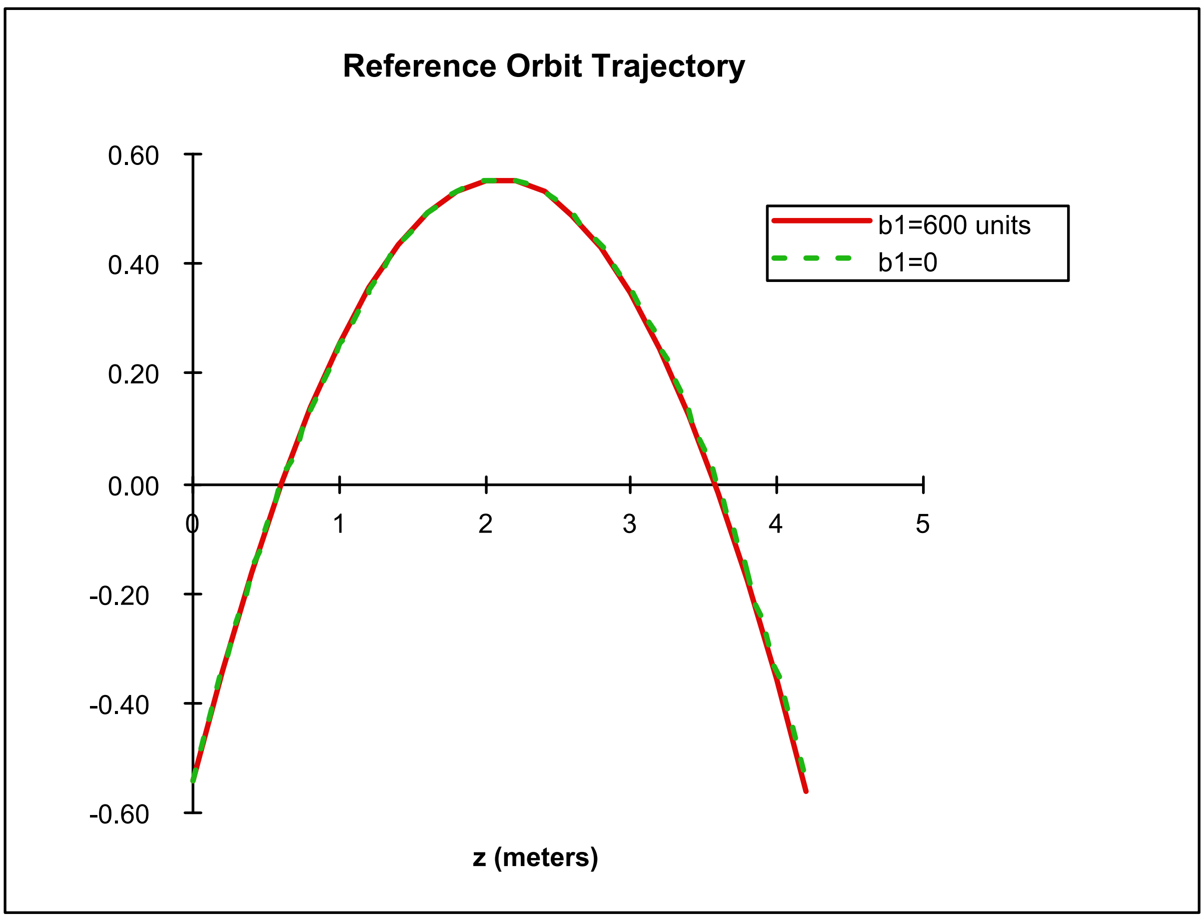

Figure 1: Reference orbit (solid) line and circular orbit (dashed line) trajectories in the 4.2 meter focusing Recycler gradient magnet.

Conclusions

The reference orbit in a combined function magnet deviates from an arc of a circle. For Recycler magnets the effect is small yet significant--the primary impact being an over (under) bending in the focusing (defocusing) magnets equivalent to approximately 40 units in strength. This effect should be compensated for in the construction or alignment of the magnets.

The deviation from the nominal bend is an example of a feed-down effect on the reference orbit in the combined function magnets of the Recycler. Analogous effects will distort the integrated focusing strength and the integrated sextupole strength. The size of these effects depends upon the magnet alignment strategy and the size of systematic higher order multipoles, and will be discussed in MI-0196.